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On Implementation of Dynamic Programming for Optimal Control Problems with Final State Constraints

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Outline

- Introduction & Motivation
- Formulation Optimal Control Problem
- Dynamic Programming
- Numerical Issues
- Method: Boundary line DP
- Results: Study on parallel electric hybrid vehicle

Introduction & Motivation

- Active research on energy management for HEV‘s
- Dynamic Programming often used for benchmarking
- Comparison of fuel consumption only valid if energy in buffers is sustained → precision of final state is crucial!
- New method allowing for precise final state

Formulation Optimal Control Problem

Find the optimal control sequence
$\pi^o(x_0) = \{u_0,\ u_1,\ \ldots,\ u_{N-1}\}$,
minimizing the cost function $J(x_0)$.

General problem

$$J(x_0) = g_N(x_N) + \sum_{k=0}^{N-1} g_k(x_k, u_k, w_k)$$

subject to:

$$\begin{aligned} x_{k+1} &= f(x_k, u_k, w_k) + x_k \\ x_0 &= x(t=0) \\ x_N &\in \mathcal{X}_N \\ x_k &\in \mathcal{X}_k \\ u_k &\in \mathcal{U}_k \end{aligned}$$

Hybrid electric vehicle

$$J(x_0) = \sum_{k=0}^{N-1} \Delta m_f(u_k, w_k)$$

subject to:

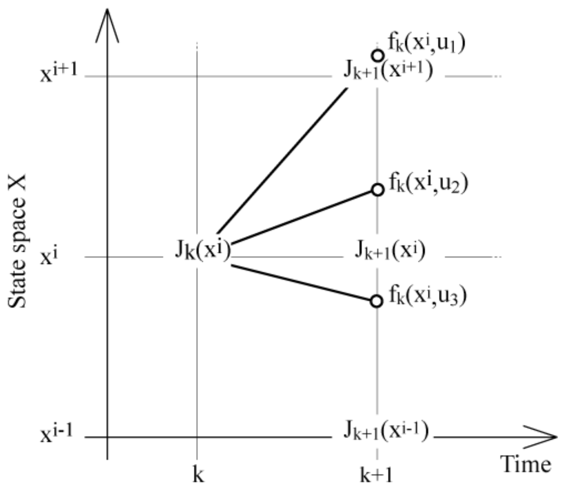
$$\begin{aligned} x_{k+1} &= f(x_k, u_k, w_k) + x_k \\ x_0 &= 0.55 \\ x_N &\geq 0.55 \\ x_k &\in [0.4,\ 0.7] \\ u_k &\in [u_{min},\ u_{max}] \end{aligned}$$

Dynamic Programming – Why for HEV's?

- Global optimum as a benchmark
- Low order of the energy management problem
- State constraints (battery state-of-charge)
- Input constraints (power/torque constraints)
- Non-linear, time-variant model

Dynamic Programming – Algorithm

Idea: Discretize state-space and proceed backward in time ($k = N, ..., 0$) to evaluate optimal cost-to-go and the corresponding control sequence at every node x_k.



Source:
Vehicle Propulsion Systems, 2nd Edition
L. Guzzella and A. Sciarretta

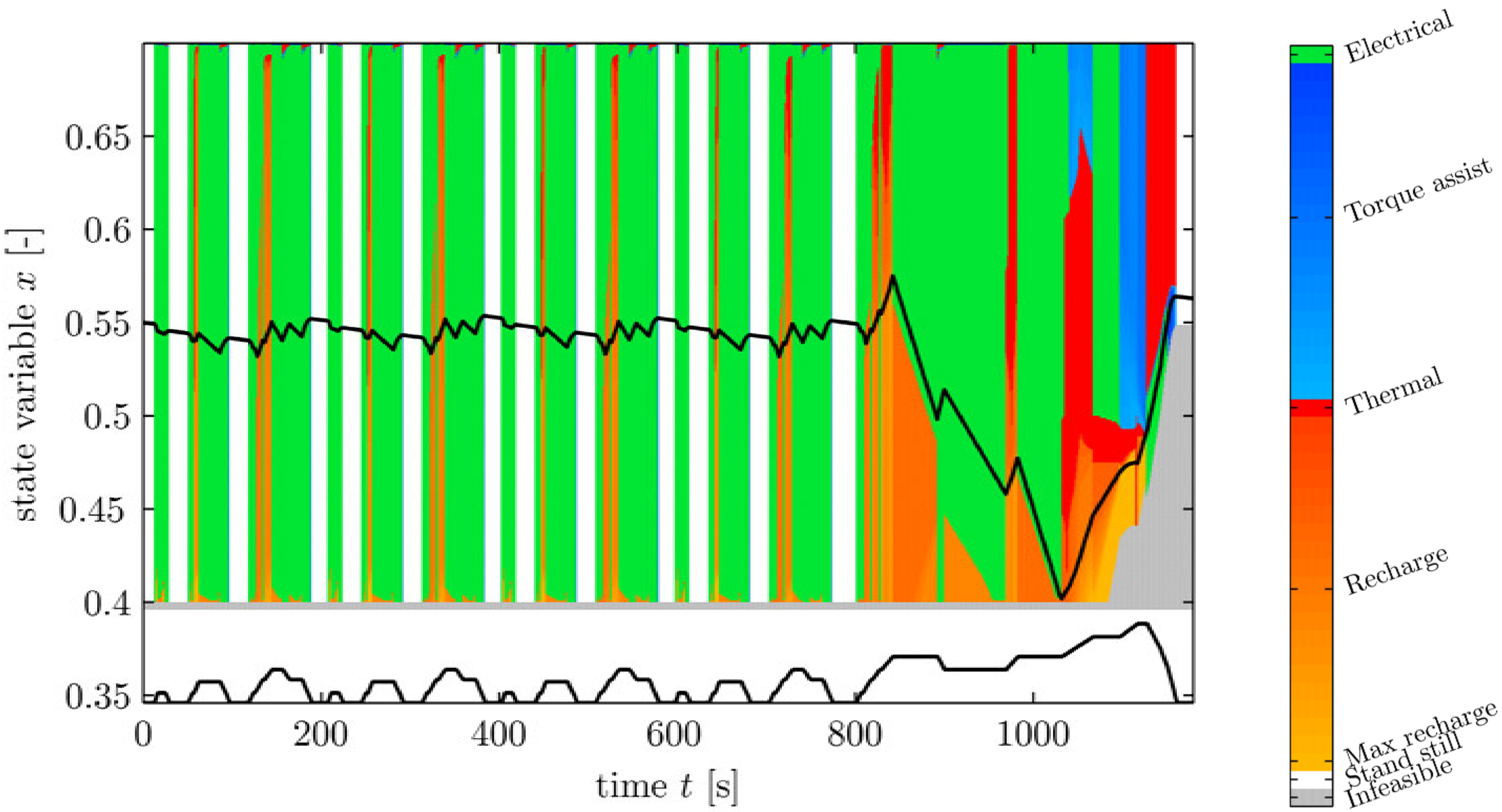
1. Assign final cost: $\mathcal{J}_N^o(x_N) = g_N(x_N)$
2. Proceed backwards by one timestep
3. For every point in the discretized state-space search the optimal input:

$$\mathcal{J}_k^o(x_k) = \min_{u_k \in U_k(x_k)} \left(g_k(x_k, u_k, w_k) + \mathcal{J}_{k+1}^o\left(f_k(x_k, u_k, w_k)\right)\right)$$

4. go to step 2 until $k = 0$ is reached

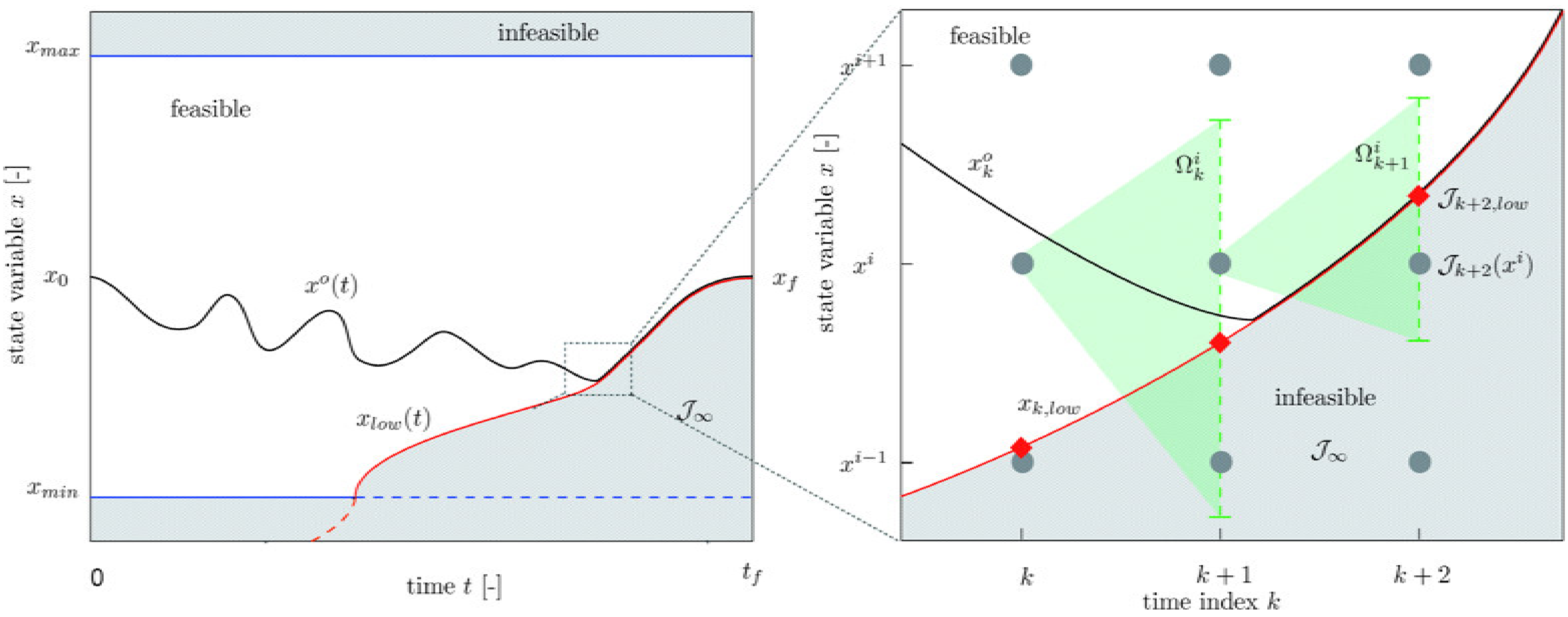
Numerical Issues

- „Optimal“ control input map with problem



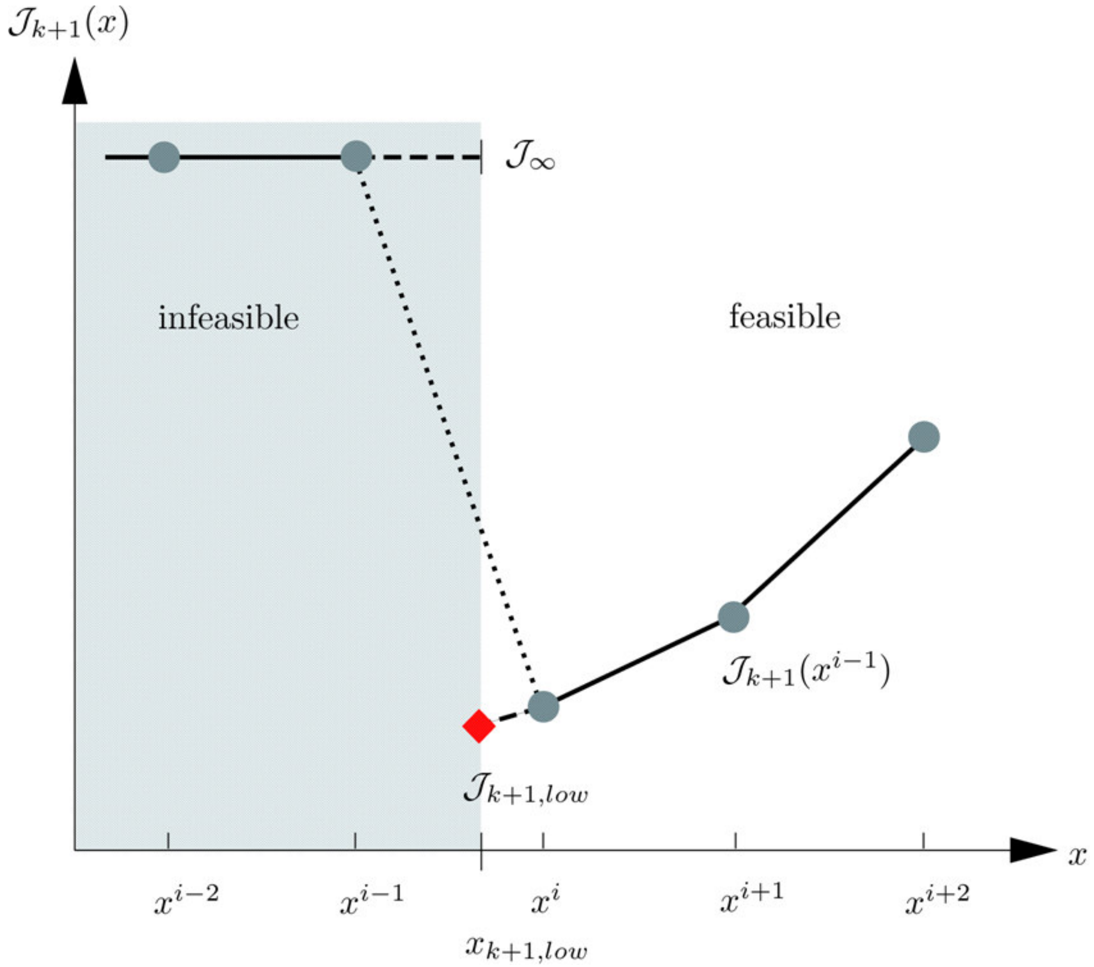
Method – Boundary Line DP

- Problem: state-trajectory close to boundary



Method – Boundary Line DP

- Interpolation of cost-to-go at the boundary



Method – Boundary Line DP

1) Evaluation of boundary line

a) Generally: iterative evaluation necessary

$$\min_{x_{k,low},u_k} \quad x_{k,low}$$
$$\text{s.t.}$$
$$f_k(x_{k,low},u_k)+x_{k,low}=x_{k+1,low}$$
$$u_k\in\mathcal{U}_k$$
$$x_{k,low}\in\mathcal{X}_k$$

b) Inversion of state dynamics possible: solving backward

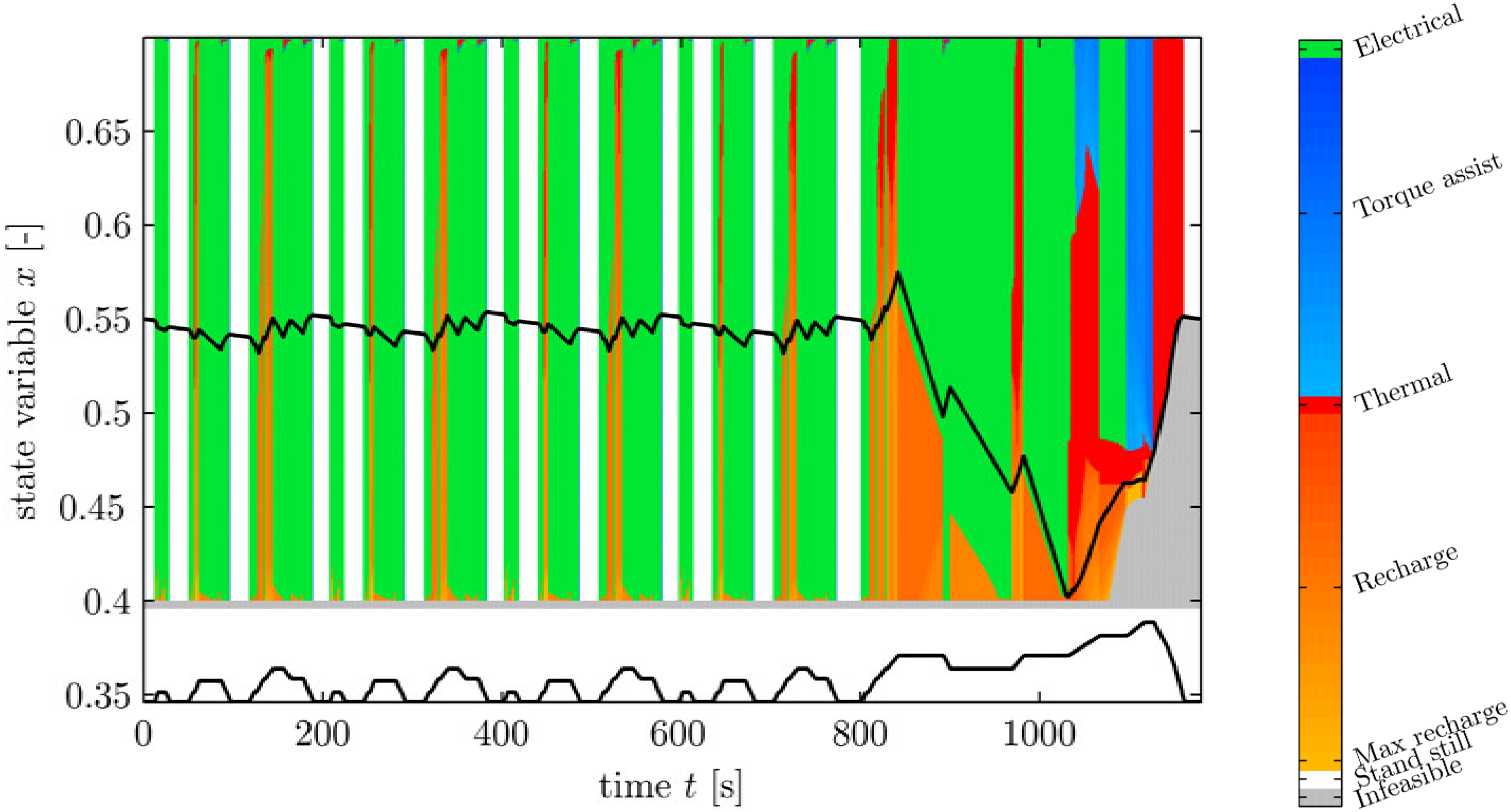
2) Application of boundary line in DP

a) Augmentation of state-grid with boundary line

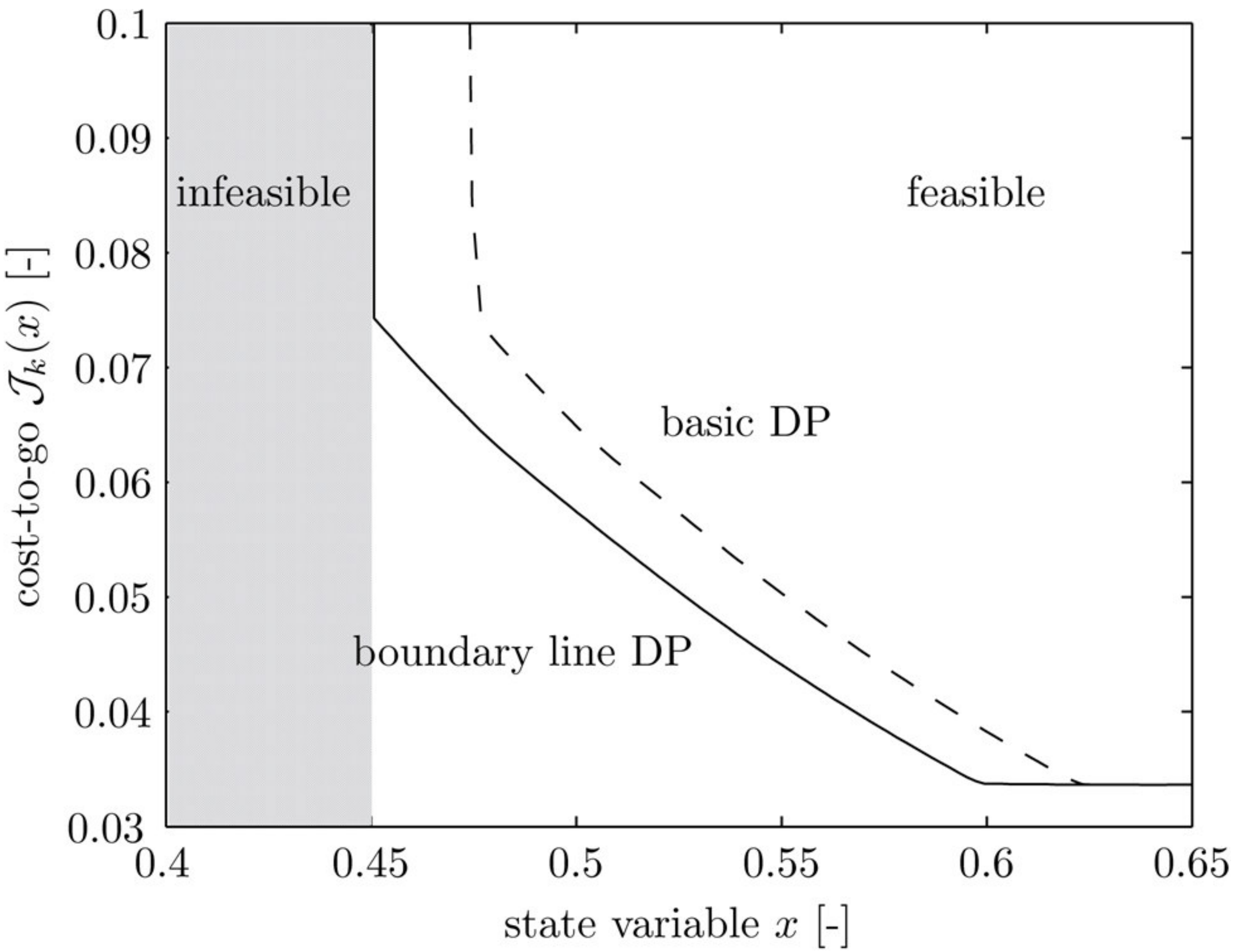
b) Adapted interpolation

Results – Optimal Control Input Map

- Optimal state-trajectory exactly on boundary line



Results – Cost-to-Go (t = 1100s)



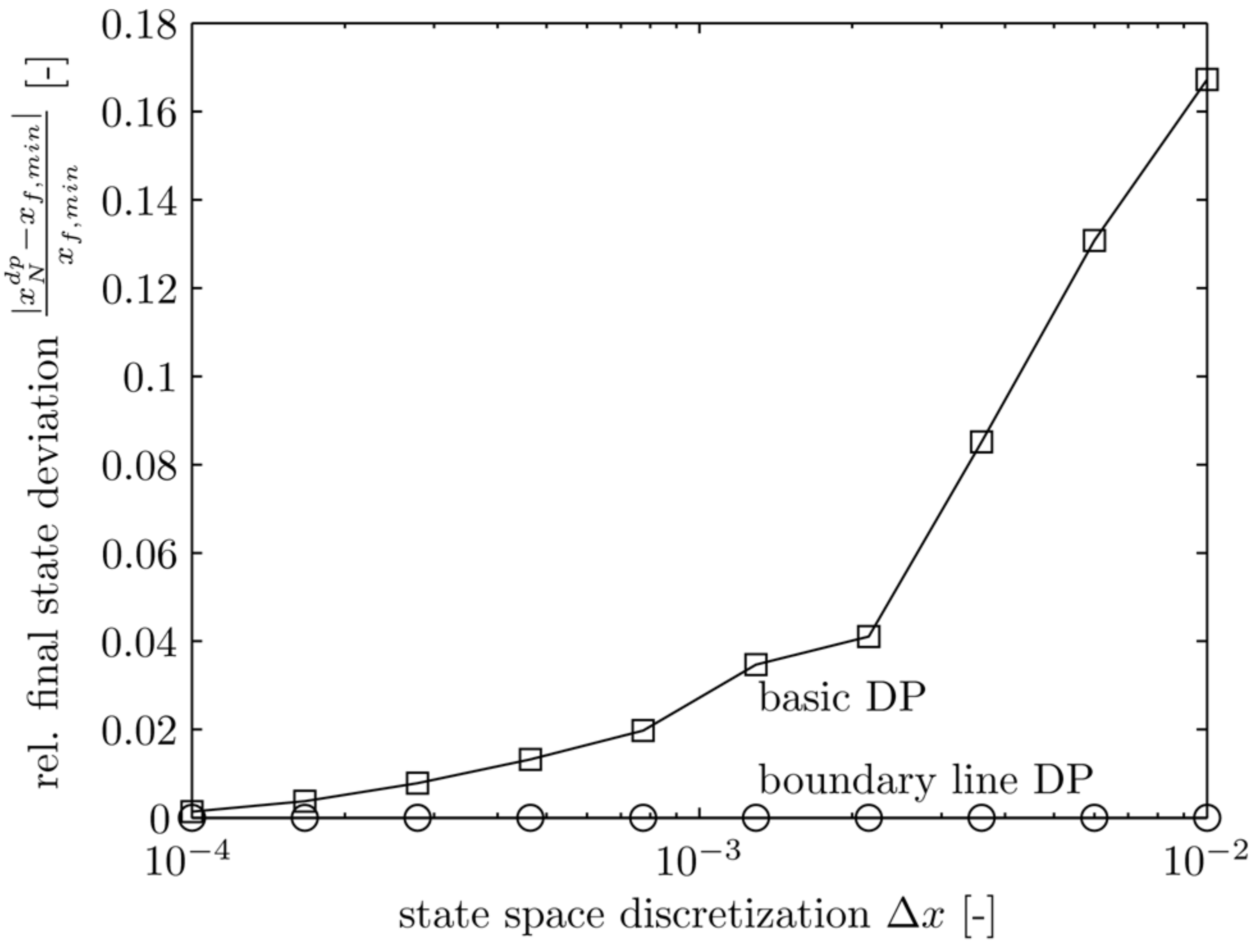
- **Basic DP**

  Cost-to-go is influenced by interpolation with infeasible states

- **Boundary line DP**

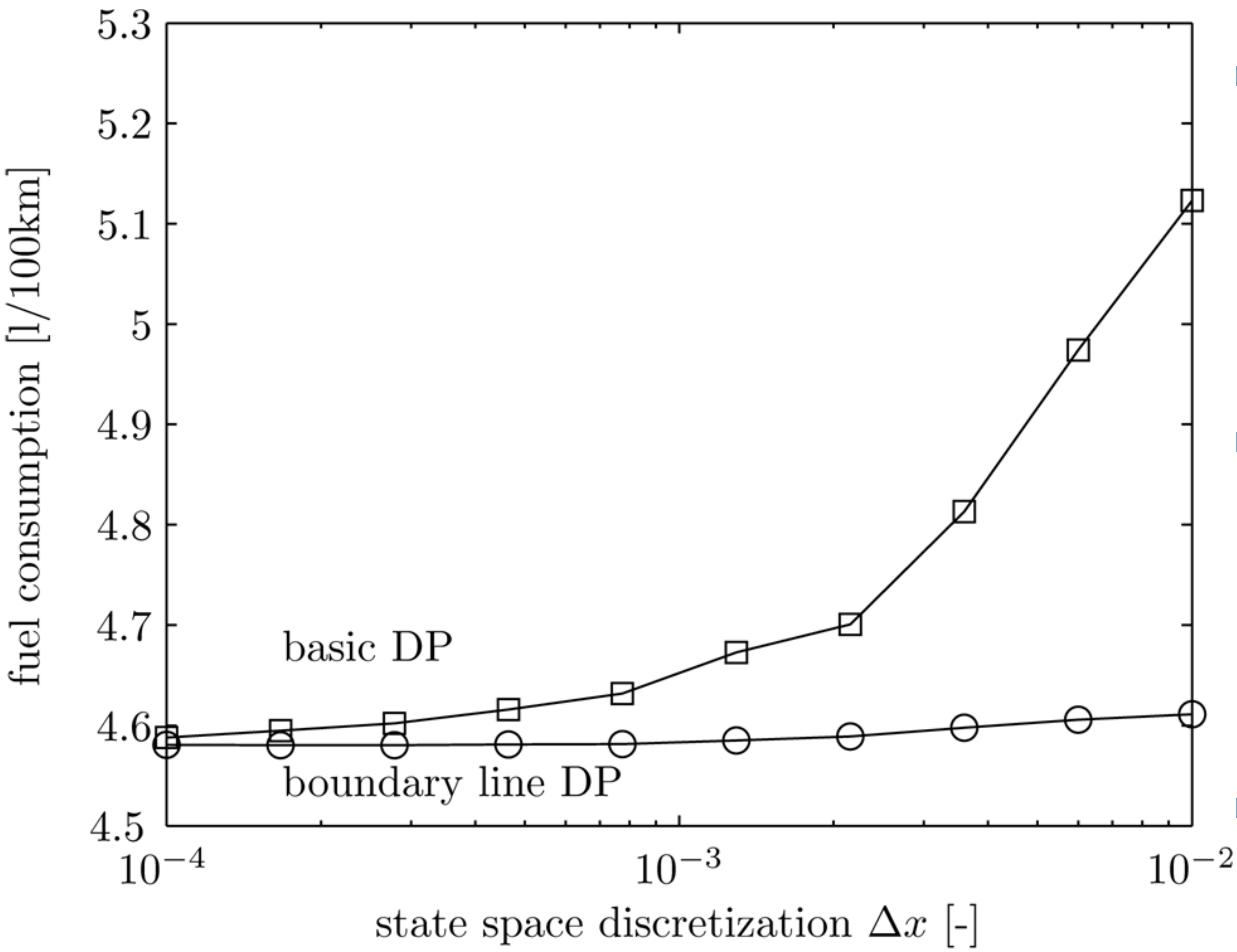
  Cost-to-go tends to infinity exactly on the boundary (by construction)

Results – Final State Deviation



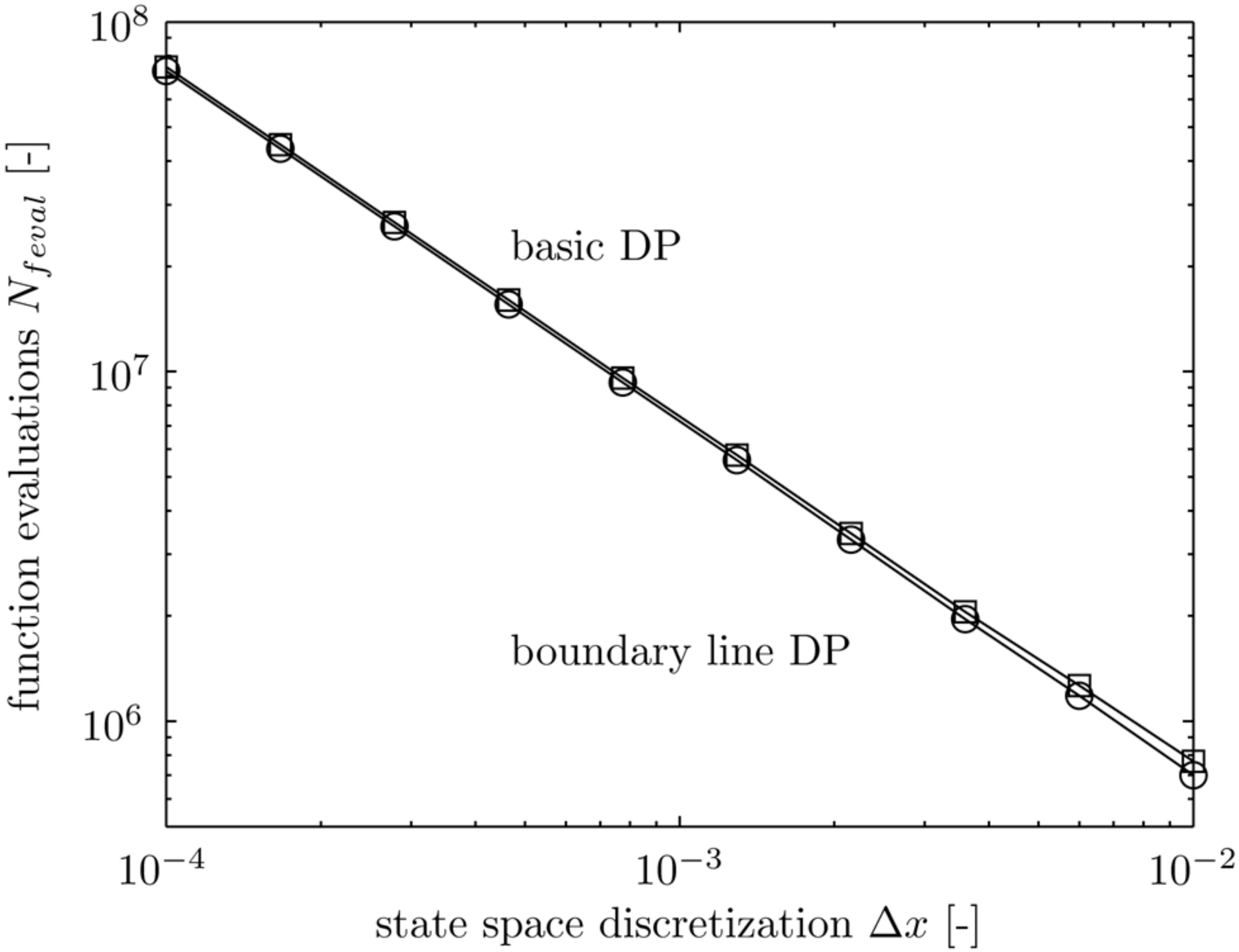
- **Basic DP**
  High resolution needed to achieve optimal final state
- **Boundary line DP**
  Deviation negligible

Results – Fuel Consumption



- **1st Effect**
  Excess fuel related to final SoC
- **2nd Effect**
  Nonlinearities in cost-to-go
- **Boundary line DP**
  Not affected from 1st effect

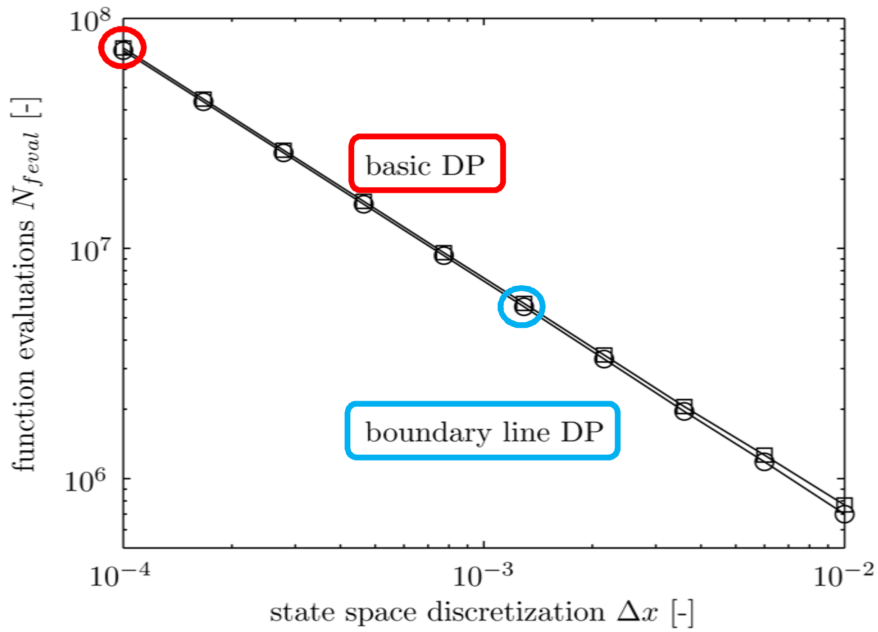
Results – Function Evaluations I



- Resolution is most important
- Reduction by omitting unfeasible region

Results – Function Evaluations II

- Computational cost at comparable fuel consumption



		basic DP	boundary line DP	ratio ($\frac{\text{basic}}{\text{b'line}}$)
state space discretization [-]	Δx	10^{-4}	$1.3 \cdot 10^{-3}$	0.0769
fuel mass consumed [kg]	J	0.3790	0.3788	1.0005
fuel consumption [l/100km]		4.5879	4.5852	1.0005
function evaluations [-]	N_{feval}	74364780	5578935	13.3296

Conclusions

- **New method improves**
  - Accuracy on final state: deviation negligible for HEV
  - Computational burden: reduced by factor 13 for HEV
- **Applicable for**
  - Optimal control problems with final state constraints
  - 1-D state-space (future work: extension)
- **Allows for**
  - Application of DP for multi-parametric optimization of component dimensions for HEV's
  - Application of DP in real-time control