1-3.2.4 Effect of differential pressure on velocities: Hertz coefficient

A) Definition of the Hertz coefficient

In rocks, the elastic wave velocities depend on the differential pressure (P_{diff}) applied to the sample. The differential pressure is equal to the difference between the confining pressure (P_{conf}) and the pore pressure (P_{pore}). In some cases, this pressure effect is very strong. The reason for this dependence is well known [Birch 1960, Walsh and Brace 1966]: it is mainly due to the ubiquitous mechanical microdefects (microcracks, grain contacts, etc.) which close under the effect of differential stress, thereby increasing the rigidity of the material.

Experimentally, we observe that the velocity *vs.* differential pressure relation obeys a power type law, at least over a limited pressure interval. Figure 1-3.20 shows that the

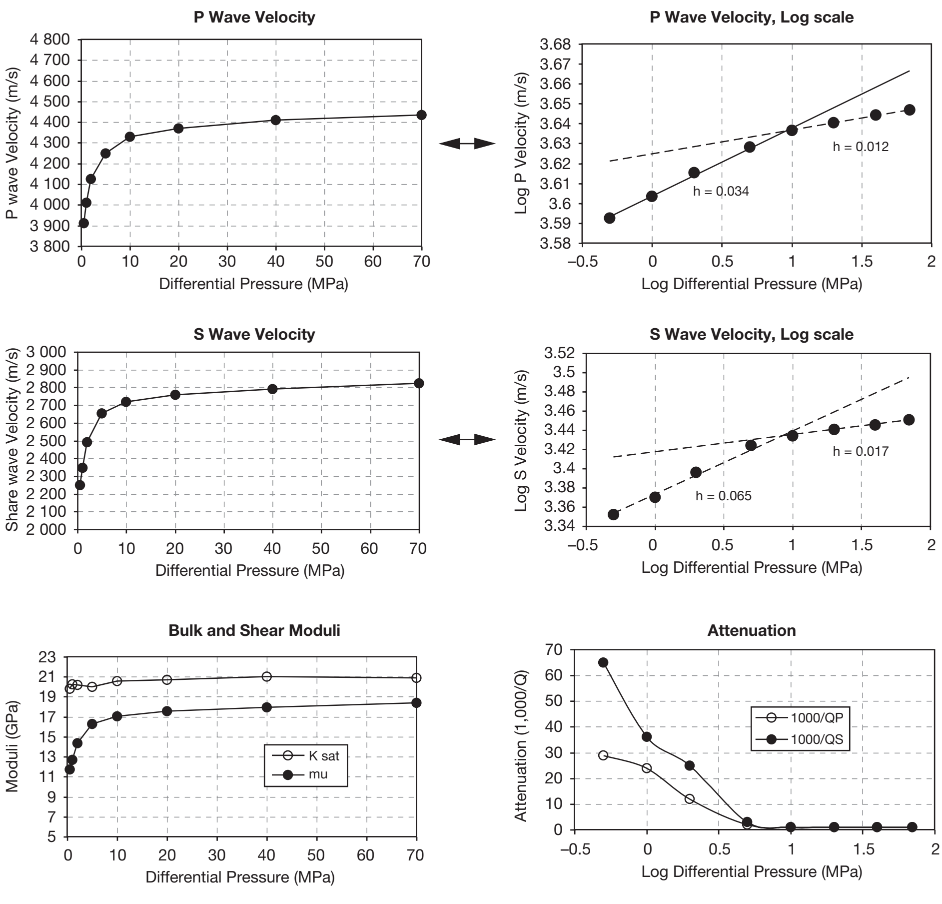

Figure 1-3.20 Effect of differential pressure on the velocities, moduli and ultrasonic attenuations of Fontainebleau sandstone ($\phi = 0.206$, $K_{air} = 2900$ mD)

Linear scale (on the left), Logarithmic scale (on the right).

formula $V = kP^h$ is perfect for the P waves and very satisfactory for the S waves, as long as pressures less than and greater than 10 MPa are considered separately. Note that the change in the rock behaviour around 10 MPa is also highly visible when we consider the attenuation (expressed in 1 000/Q). Above 10 MPa, attenuation is too low to be estimated using the spectral ratio method. In this clean sandstone, attenuation and velocity variation both have the same cause: the grain contacts (an extreme example is shown Fig. 1-2.16, p. 148) which are virtually all closed under a differential pressure of 10 MPa. This limit varies considerably depending on the rock type. In Vosges sandstone with low clay content, this limit is above 40 MPa.

Hertz demonstrated by calculation that in a packing of isodiametral spheres, in elastic contact, the velocity varied with pressure according to a power function and that for the P waves, the exponent was 1/6. The exponent h is therefore known as the "Hertz coefficient".

To evaluate the effect on wave velocities of a differential pressure variation in a reservoir rock, we simply need to know the Hertz coefficient. It is then easy to calculate the proportionality factor of the power function and apply it to the values supplied by sonic logging.

B) Hertz coefficient measurement and core measurement representativeness

It is extremely easy to measure the Hertz coefficient on sample: simply measure the velocity variation under the effect of the differential pressure and calculate the gradient of the linear relation V *vs.* P_{diff} using bilogarithmic axes. The most important point is to accurately measure a relative velocity variation, which corresponds to the simplest case.

To our knowledge, no method is available to measure the Hertz coefficient *in situ*. We are therefore forced to rely completely on laboratory measurements for which there is a problem of core mechanical representativeness.

During coring operations, the rock undergoes sudden stress variations (relaxation) (see § 1-1.1.6, p. 41). The mechanical damage suffered by the rock may be increased by alteration of some minerals (especially clays) due to drying. We may therefore legitimately question the mechanical representativeness of the cores. It is considered that returning the cores to the confining pressure (sometimes to values greater than those found in the reservoir) may compensate this effect for velocities, for example. What is the situation, however, as regards the Hertz coefficient which precisely characterises the sensitivity to pressure?

Some experimental studies [Ness *et al*., 2000] demonstrate this damage to the samples. We tend towards a similar conclusion by statistically comparing the values of the Hertz coefficient measured on cores and on outcrop samples. The outcrop samples have undergone very slow decompression (over geological time scale) and have therefore been protected from this cause of damage. A long weathering process started, however, on approaching the surface.

The histograms of Figure 1-3.21 show that, for sandstone samples, the Hertz coefficients are statistically much larger for core samples than for outcrops, both for P waves and S waves.

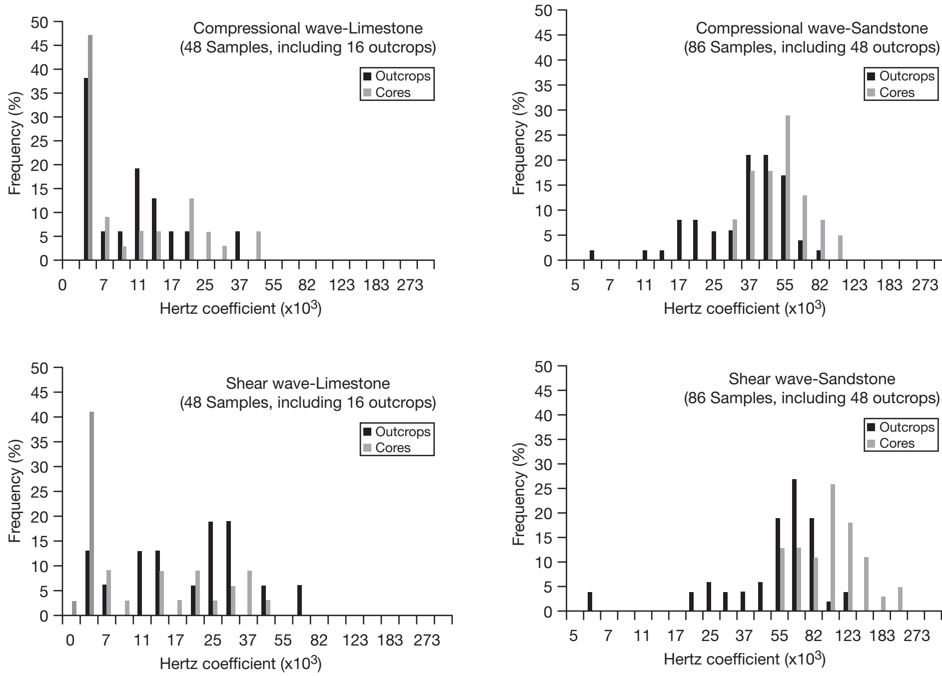

Figure 1-3.21 Distribution of Hertz coefficients in sandstones and limestones. Samples from outcrops and cores

C) Values of Hertz coefficients in rocks, the limestone-sandstone contrast

There is a striking contrast between the Hertz coefficients of limestone and sandstone. Figure 1-3.21 shows that the Hertz coefficient for more than $^3/_4$ of the limestones reported is less than that of the sandstones, for both P and S waves, to such an extent that there is practically no overlap between the histograms. This is a quantitative illustration of the well-known general observation: velocities in outcrop limestones are much less sensitive to differential pressure than in sandstones.

The reason is quite simple: in limestones, mechanical microdefects are much less frequent than in sandstones, since calcite cement forms much more rapidly. Not enough data are available on dolomites to obtain a statistical value, but the Hertz coefficients seem much higher in sucrosic dolomites (intercrystalline contacts) than in the other carbonate rocks.

Note on Figure 1-3.20 that differential pressure has virtually no effect on the bulk modulus K_{sat} of Fontainebleau sandstone. The slight fluctuations at low pressures lie within the margin of experimental uncertainty. The effect of differential pressure corresponds entirely to the variation in shear modulus. This is frequently observed in clean and very low clay content sandstones. In shaly sandstones as such, however, the bulk modulus may vary considerably with differential pressure. This – at least partial – insensitivity of K_{sat} to the differential pressure explains the distribution of HertzS/HertzP ratios in sandstones, where the modal value is about 1.5.

D) Notion of terminal velocity

This dependence of velocity on pressure (sometimes very high) poses a practical problem: what differential pressure state should be chosen to define the characteristic velocity of a rock in order to determine velocity/porosity relations, for example? In practice, we use the notion of terminal velocity, based on a number of converging observations. Firstly, the relation between velocity and pressure is such that, for high pressures, the relative velocity increase drops sharply. For geophysical applications, the maximum differential pressures exerted on porous rocks are in the region of a few tens of MPa. If a porous rock is subjected to excessive differential pressure, the structure may be destroyed by the pores being crushed (§ 1-1.1.6A, p. 42). This phenomenon is quite clear in some limestones. For each rock therefore, there is a "limiting" upper velocity known as the "terminal" velocity. This velocity will be used to characterise the material.

Although the definition may appear shocking due to its lack of precision, it nevertheless remains a useful example of pragmatism to characterise the properties of consolidated rocks. This notion is of no use, however, if the material undergoes significant variations under the effect of even moderate stress. This is the case with clays or surface sediments undergoing mechanical "consolidation".

1-3.2.5 Effect of the saturating fluid: Gassmann equation and its linear approximation as a function of K_{fl}

A) Biot-Gassmann equation

a) Complete expression

Numerous publications have been dedicated to the Biot-Gassmann equation, used to quantify the effect of a variation in saturating fluid on seismic velocities. For a detailed study, readers may refer to the general books mentioned above [Bourbié *et al.*, 1987, Mavko *et al.*, 1998]. We will restrict ourselves to a brief summary.

Gassmann [1951] proposed an explicit expression of Biot's macroscopic parameters in terms of petrophysical parameters of direct practical interest to calculate the effect of fluid on velocities. We consider two different states for a fluid-saturated porous medium:

- the drained or dry state in which the pressure of the saturating fluid remains unchanged during the mechanical stress (local variations in fluid volume induced by the vibratory deformation being "compensated" by the outside);
- the non-drained or saturated state in which the local variations in fluid content are zero during the mechanical stress. Some of the vibratory stress will be taken up as a pressure variation in the saturating fluid.

Gassmann calculated, for a virtually static stress, the difference in elastic modulus between a drained porous medium and the same medium in non-drained state. This theory assumes a continuous and homogeneous porous medium, but implies no conditions on the pore geometry.